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Non-eikonal corrections for the scattering of spin-one particles

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Abstract. The Wallace Fourier-Bessel expansion of the scattering amplitude is generalised to the case of the scattering of a spin-one particle from a potential with a single tensor coupling as well as central and spin-orbit terms. A generating function for the eikonal-phase (quantum) corrections is evaluated in closed form. For medium-energy deuteron-nucleus scattering, the first-order correction is dominant and is shown to be significant in the interpretation of analysing power measurements. This conclusion is supported by a numerical comparison of the eikonal observables, evaluated with and without corrections, with those obtained from a numerical resolution of the Schrödinger equation for d-^{58}Ni scattering at incident deuteron energies of 400 and 700 MeV.

PACS. 03.65.Nk Scattering theory – 24.70.+s Polarization phenomena in reactions – 25.10.+s Nuclear reactions involving few-nucleon systems – 25.40.Hs Transfer reactions

1 Introduction

The eikonal approximation developed by Glauber [1] is one of the most successful and transparent theories that describe the scattering of hadrons from nuclei at small angles and high energies. There have been attempts to extend the Glauber approximation to include tensor forces in spin-half [2] as well as spin-one interactions [3]. However, the Glauber method of deriving the eikonalised amplitude is not directly applicable to the case of spin-one particles interacting with spinless targets because of the non-commuting nature of the tensor potential. As will be shown, this problem can be avoided in the case where just one of the three tensor potentials is present, and that is the situation that is studied in detail here.

The standard Glauber amplitude exhibits some deficiencies in the multiple-scattering terms [4]. While the first-order term in the interaction power series expansion is real, and identical to that of the first-order Born approximation, the higher-order terms alternate between being purely imaginary and purely real. In particular, while the imaginary part of the second-order Born approximation has a corresponding Glauber term, the real part disappears within the Glauber expansion. This defect can be traced back to the assumption that the phase is evaluated by integrating the potential along a classical trajectory, which is taken to be a straight line.

Wallace [5–7] has developed a complete high-energy expansion of the Fourier-Bessel representation of the scattering amplitude for the scattering of a non-relativistic spin-zero particle from a central potential. This relaxes the original small-angle assumption and therefore improves upon the Glauber straight-line approximation. Wallace's method works by converting the partial-wave scattering amplitude exactly into a Fourier-Bessel integral representation in terms of the impact parameter b. By expanding the WKB phase as a series in powers of the potential strength, to first order he obtains the Glauber eikonal phase. Higher-order terms then provide the leading quantum corrections to the straight-line semiclassical path assumption. Among the other benefits, these corrections induce a real part in the second-order contribution to the scattering amplitude. Wallace's work has recently been extended to few-body Glauber models in which the projectile is treated as a composite system of clusters in which the eikonal phase for the scattering of each projectile constituent from the target is corrected [8,9]. However, only central potentials were considered.

Waxman *et al.* [10] extended this scheme to investigate the scattering of non-relativistic spin-half particles. In their work they distinguished carefully between the impact

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parameter dependence coming from linear-momentum factors and the angular-momentum variation of the eikonal phases arising from the spin-orbit coupling. This distinction is important in the case of spin-dependent interactions, which are momentum dependent as well as lacking spherical symmetry. Waxman *et al.* also emphasised that for spin-half scattering, due to parity conservation, the Schrödinger equation does not mix states with total angular momentum $j = l + 1/2$ and $j = l - 1/2$, making it possible to evaluate the eikonal quantum corrections unambiguously.

We generalise the approach of Waxman *et al.* [10] to the scattering of spin-one particles. A deuteron may interact with a spinless target through a central, a vector spin-orbit and three tensor potentials, corresponding to the coupling of the spin to the radial variable, the linear momentum, and the orbital angular momentum of the particle [11]. The relative importance of these potentials in the scattering of deuterons from nuclei depends very much upon the energy range being studied. In order to avoid the commutation problem, in this paper we limit our analysis to the second-order spin-orbit tensor term V_L, which contains operators that do not mix different angular-momentum states. After presenting the partial-wave structure of the amplitudes in sect. 2, Wallace's derivation of the Fourier-Bessel representation for spinless interactions is discussed in sect. 3 and then extended to spin-one scattering. The expansion of the WKB phase shifts in powers of the potential strength is discussed in sect. 4, using the methods derived by Waxman *et al.* for spin-half scattering. The first quantum corrections to the eikonal phases, deduced in sect. 5, improve the second-order eikonal contributions, bringing the amplitudes appreciably closer to the high-energy limit of the Born series. The accuracy of the modified eikonal approach is tested in sect. 6 by resolving numerically a relativistic Schrödinger equation describing elastic scattering of deuterons by nuclei. Observables corresponding to d-^{58}Ni scattering at 400 and 700 MeV are then compared in the numerical and eikonal approaches and it is seen that the quantum modifications improve significantly the agreement between the two. Our conclusions are presented in sect. 7.

2 Partial-wave decomposition

In the case of spin-half scattering, parity and angular-momentum conservation ensure that states of different orbital angular momentum do not mix, but in general this is no longer true for spin-one particles. There are three possible tensor potentials proportional to the tensors

$$\begin{aligned} T_L &= \mathbf{S}_2 \cdot \mathbf{R}_2 \left(\mathbf{L} \cdot \mathbf{L}\right) = \left(\mathbf{S} \cdot \mathbf{L}\right)^2 + \mathbf{S} \cdot \mathbf{L} - \frac{2}{3} L^2 , \\ T_r &= \left(\mathbf{S} \cdot \mathbf{r}\right)^2 - \frac{2r^2}{3} , \\ T_p &= \left(\mathbf{S} \cdot \mathbf{p}\right)^2 - \frac{2p^2}{3} . \end{aligned} \tag{2.1}$$

We have here used the recoupling operator for spin S

$$\begin{aligned} \mathbf{S}_2 \cdot \mathbf{R}_2 \left(\boldsymbol{\alpha} \cdot \boldsymbol{\beta}\right) = &\left(\mathbf{S} \cdot \boldsymbol{\alpha}\right) \left(\mathbf{S} \cdot \boldsymbol{\beta}\right) \\ &- \frac{i}{2} \mathbf{S} \cdot \left(\boldsymbol{\alpha} \times \boldsymbol{\beta}\right) - \frac{S^2}{3} \left(\boldsymbol{\alpha} \cdot \boldsymbol{\beta}\right) , \end{aligned} \tag{2.2}$$

in conjunction with the position r, momentum p, and angular-momentum L operators. Of the three tensor operators, only the quadratic spin-orbit term has the simplifying feature of not mixing the angular-momentum states and it will be the only one studied in this paper. We therefore consider the optical potential defined by [11],

$$V\left(\mathbf{S}, r\right) = V_C\left(r\right) + V_S\left(r\right)\ \mathbf{S} \cdot \mathbf{L} + V_L\left(r\right)\ T_L , \tag{2.3}$$

where we are working in units for which $2m = \hbar = 1$. The resulting scattering amplitude has the partial-wave decomposition

$$\begin{aligned} F\left(\theta\right) = &\frac{1}{2ik} \sum_{l=0}^{\infty} \left(2l+1\right) \\ &\times \left\{ T_l^+ \prod_{l+} + T_l^- \prod_{l-} + T_l^0 \prod_{l0} \right\} P_l\left(\cos\theta\right) , \end{aligned} \tag{2.4}$$

where the partial-wave amplitude is expressed in terms of the phase shift δ_l^j through $T_l^j = \left(e^{2i\delta_l^j} - 1\right)$, and the projection operators are defined by [12]

$$\begin{aligned} \prod_{l+} &= \frac{\left(\mathbf{S} \cdot \mathbf{L}\right)^2 + \left(l+2\right) \left(\mathbf{S} \cdot \mathbf{L}\right) + \left(l+1\right)}{\left(l+1\right) \left(2l+1\right)} , \\ \prod_{l-} &= \frac{\left(\mathbf{S} \cdot \mathbf{L}\right)^2 + \left(l-1\right) \left(\mathbf{S} \cdot \mathbf{L}\right) - l}{l \left(2l+1\right)} , \\ \prod_{l0} &= -\frac{\left(\mathbf{S} \cdot \mathbf{L}\right)^2 + \left(\mathbf{S} \cdot \mathbf{L}\right) - l\left(l+1\right)}{l\left(l+1\right)} . \end{aligned} \tag{2.5}$$

Here the indices $(+, 0, -)$ denote states with total angular momentum $j = (l+1, l, l-1)$, respectively.

Following Glauber [1], we take the z (quantisation) axis to lie along the direction of the average of the initial and final momenta $\mathbf{k} = (\mathbf{k}_\mathrm{i} + \mathbf{k}_\mathrm{f})$, so that the experimental observables must be obtained by subsequently rotating the scattering matrix through an angle of $-\theta/2$ around the normal $\hat{\mathbf{n}}$ to the scattering plane, where θ is the scattering angle. In the Glauber frame the scattering matrix can be written as

$$\begin{aligned} F\left(\theta\right) = &A\left(\theta\right) + B\left(\theta\right)\ \left(\mathbf{S} \cdot \hat{\mathbf{n}}\right) \\ &+ C_k\left(\theta\right)\ \mathbf{S}_2 \cdot \mathbf{R}_2 \left(\hat{\mathbf{k}}, \hat{\mathbf{k}}\right) \\ &+ C_n\left(\theta\right)\ \mathbf{S}_2 \cdot \mathbf{R}_2 \left(\hat{\mathbf{n}}, \hat{\mathbf{n}}\right) , \end{aligned} \tag{2.6}$$

where

$$A(\theta) = \frac{1}{6ik}\sum_{l=0}^{\infty}\Big\{ (2l+1)\,T_l^+ + (2l-1)\,T_l^- + (2l+1)\,T_l^0\Big\} P_l(\cos\theta)\,,$$

$$B(\theta) = \frac{1}{4k}\sum_{l=0}^{\infty}\Big\{\frac{(2l+3)}{(l+1)}T_l^+ - \frac{(2l-1)}{l}T_l^- - \frac{(2l+1)}{l\,(l+1)}T_l^0\Big\} P_l^1(\cos\theta)\,,$$

$$C_k(\theta) = \frac{1}{2ik\sin\theta}\sum_{l=0}^{\infty}\Big\{\frac{1}{(l+1)}T_l^+ + \frac{1}{l}T_l^- - \frac{(2l+1)}{l\,(l+1)}T_l^0\Big\} P_l^1(\cos\theta)\,, \tag{2.7}$$

$$C_n(\theta) = (3\cos\theta+1)\,C_k(\theta) + \frac{1}{2ik}\sum_{l=0}^{\infty}\Big\{l\,T_l^+ + (l+1)\,T_l^- - (2l+1)\,T_l^0\Big\} P_l(\cos\theta)\,.$$

Note that $T_l^- = 0$ for $l = 0$.

3 Fourier-Bessel expansion of the scattering amplitude

We here follow closely the procedure established by Wallace [5] in his study of spin-zero interactions and extended by Waxman *et al.* [10] to the treatment of the spin-half case. For the scattering of a spin-zero particle by a spherically symmetric potential, the partial-wave decomposition of eq. (2.4) reduces to

$$F(\theta) = \frac{1}{2ik}\sum_{l=0}^{\infty}(2l+1)\,\left(e^{2i\delta(l)}-1\right)\,P_l(\cos\theta). \tag{3.1}$$

The corresponding impact parameter, b, representation is

$$F(\theta) = -ik\int_0^{\infty} \mathrm{d}b\, b\, J_0(qb)\,\Gamma(b)\,, \tag{3.2}$$

where the profile function $\Gamma(b) = \left(e^{i\,\chi(b)}-1\right)$ and the momentum transfer $q = 2k\sin(\theta/2)$.

To derive a connection between these two descriptions, Wallace converted the partial-wave sum into an integral over real values of l, using the Euler-summation formula. He then expanded the Legendre polynomials as infinite series of derivatives of the Bessel functions $J_0(x)$. For a well-behaved potential, which gives rise to phase shifts that can be interpolated smoothly for real l, the eikonal phase $\chi(b)$ may be related to the $\delta(l)$ through

$$e^{i\,\chi(b)} = (2l+1)^{-1}\,\mathsf{W}\,(2l+1)\,e^{2i\delta(l)}, \tag{3.3}$$

where the operator W is given by

$$\mathsf{W}(b) = \sum_{m=0}^{n}\frac{1}{(2m)!}\,B_{2m}(x)\left\{-\frac{1}{4}(2l+1)\frac{\partial}{\partial b}\right\}\left[\frac{\partial}{\partial b}\right]^{2m}. \tag{3.4}$$

The $B_{2m}(x)$ are generalised Bernoulli polynomials [13], with $B_0(x) = 1$ and $B_2(x) = -x/6$. In the evaluation of eq. (3.3) the semiclassical identification of the impact parameter, $b = (l+1/2)/k$, is assumed.

We now extend the Fourier-Bessel representation of the scattering amplitude to spin-one interactions described by the potential of eq. (2.3). From the definition of the S-matrix elements, the profile function corresponding to the amplitude $A(\theta)$ becomes

$$\Gamma_A(b) = -\frac{i}{3}\Big\{ e^{[i(\bar\chi(b)-D\chi(b)/3)]}\times\left(1+2\cos\left(\Delta\chi(b)/2\right)\,e^{iD\chi(b)/2}\right) - 3 + \frac{2i}{kb}\sin\left(\Delta\chi(b)/2\right)\,e^{[i(\bar\chi(b)+D\chi(b)/6)]}\Big\}, \tag{3.5}$$

where the scalar, vector and tensor phase functions $\bar\chi(b)$, $\Delta\chi(b)$, and $D\chi(b)$ are related to the phase shifts by

$$\begin{aligned}\bar\chi(b) &\equiv \tfrac{2}{3}\,\left(\delta_l^+ + \delta_l^- + \delta_l^0\right)\,,\\ \Delta\chi(b) &\equiv 2\,\left(\delta_l^+ - \delta_l^-\right)\,,\\ D\chi(b) &\equiv 2\,\left(\delta_l^+ + \delta_l^- - 2\,\delta_l^0\right)\,.\end{aligned} \tag{3.6}$$

The other profile functions may be similarly treated, leading to

$$\Gamma_B(b) = \frac{1}{2\sin\theta}\,\Big\{2ikb\sin\left(\Delta\chi(b)/2\right)\,e^{[i(\bar\chi(b)+D\chi(b)/6)]} + \cos\left(\Delta\chi(b)/2\right)\left(e^{[i(\bar\chi(b)+D\chi(b)/6)]}-e^{[i(\bar\chi(b)-D\chi(b)/3)]}\right) - \frac{i}{kb}\sin\left(\Delta\chi(b)/2\right)\,e^{[i(\bar\chi(b)+D\chi(b)/6)]}\Big\}\,, \tag{3.7}$$

$$\Gamma_C^K(b) = -\frac{i}{\sin^2\theta}\,\Big\{e^{[i(\bar\chi(b)-D\chi(b)/3)]}\times\left(1-\cos\left(\Delta\chi(b)/2\right)\,e^{[iD\chi(b)/2]}\right) + \frac{i}{2kb}\sin\left(\Delta\chi(b)/2\right)\,e^{[i(\bar\chi(b)+D\chi(b)/6)]}\Big\}\,. \tag{3.8}$$

The resulting amplitudes can then be written as Fourier-Bessel transforms

$$\begin{aligned}A(\theta) &= k\int_0^{\infty} b\,\mathrm{d}b\,J_0(qb)\,S_F(b)\,\Gamma_A(b)\,,\\ B(\theta) &= 2\sin(\theta/2)\int_0^{\infty} b\,\mathrm{d}b\,J_1(qb)\,S_F(b)\,\Gamma_B(b)\,,\\ C_k(\theta) &= 2\sin(\theta/2)\int_0^{\infty} b\,\mathrm{d}b\,J_1(qb)\,S_F(b)\,\Gamma_C^k(b)\,,\\ C_n(\theta) &= \frac{1}{2}(3\cos\theta+1)\,C_k(\theta) - k\sin^2\theta\int_0^{\infty} b\,\mathrm{d}b\,J_0(qb)\,S_F(b)\,\Gamma_C^k(b)\,.\end{aligned} \tag{3.9}$$

The operator $S_F(b)$, defined by,

$$S_F\,(b) = b^{-1}\,\mathsf{W}\,(b)\;b. \tag{3.10}$$

is the impact parameter version of eq. (3.3).

4 A dynamical model for the phase shift

Following Wallace [5], we take the WKB approximation and its generalisations as the dynamical model for the phase shift function:

$$\delta_j^{\rm WKB}\,(l) = (l+1/2)\,\frac{\pi}{2} - kr_t - \int_{r_t}^{\infty} \left\{ \left(k^2 - V_j\,(l,r) - (l+1/2)^2/r^2\right)^{1/2} - k \right\} {\rm d}r. \tag{4.1}$$

For a given incident momentum (k), the distance of closest classical approach (r_t) for a particle of angular momentum l gets close to that of the free case (l/k) as l becomes large. It should be noted that this formula should remain valid even when the potential depends explicitly upon l.

Expanding the integrand in powers of $V(l,r)/k^2$ and integrating by parts, the first-order term gives the standard eikonal-phase function

$$\delta_j^{\rm WKB}\,(l)\,|_{\rm 1st\,order} \cong -\frac{1}{4k}\int_{-\infty}^{\infty} V_j\,(l,r)\;{\rm d}z, \tag{4.2}$$

where $r^2 = b^2 + z^2$. The impact parameter in this limiting case is the distance of closest approach of the classical trajectory, though this interpretation becomes fragile as l approaches zero.

In the neighbourhood of a turning point the expansion parameter becomes too large to be at all meaningful. Wallace [5–7] expands eq. (4.1) around the dimensionless parameter $\varepsilon_0 \sim V_0/\hbar kv$, (where V_0 is the potential strength, and v is the incident velocity) and shows that the WKB phase shift can be expressed as

$$\delta_j^{\rm WKB}\,(b) = \sum_{n=0}^{\infty} \delta_n^j\,(b)\,, \tag{4.3}$$

where

$$\delta_n^j\,(b) = -\frac{1}{2k\,(n+1)\,!}\left\{\left\{\frac{b}{k}\frac{\partial}{\partial b} - \frac{\partial}{\partial b}\right\}\frac{1}{2k}\right\}^n \times \int_{r_t}^{\infty} {\rm d}z\;[V_j\,(b\,,\,r)]^{\,n+1}. \tag{4.4}$$

Waxman *et al.* [10] noticed that the differential operator appearing here could be replaced by

$$-\left\{\frac{\partial}{\partial k}\right\}_l = \frac{b}{k}\left\{\frac{\partial}{\partial b}\right\}_k - \left\{\frac{\partial}{\partial k}\right\}_b. \tag{4.5}$$

This fixed-l form of the operator demonstrates the independence of the expansion in eq. (4.4) from the l-structure of the potential and also stresses the dynamical nature of the series. Using the displacement operator, they then rewrite eq. (4.1) as

$$\delta_j^{\rm WKB}\,(l) = \int_{k^2}^{\infty} {\rm d}k'^2 \int_0^{\infty} r{\rm d}r \left\{1 - \exp\left[-V_j\,(l,r)\left(\frac{\partial}{\partial k'^2}\right)_l\right]\right\} \times \left(k'^2 r^2 - (l+1/2)^2\right)^{-1} \Theta\,(k'r - (l+1/2))\,, \tag{4.6}$$

where Θ is the Heaviside step function. Expanding the exponential and integrating over k'^2, it is then straightforward to arrive at the Wallace expansion. The first two terms of the series are

$$\begin{aligned} \delta_0^j\,(b) &= -\frac{1}{2k}\int_{-\infty}^{\infty} {\rm d}z\,V_j\,(\,r\,)\,, \\ \delta_1^j\,(b) &= -\frac{1}{8k^3}\left\{1 + b\frac{\partial}{\partial b} - k\frac{\partial}{\partial k}\right\}\int_{-\infty}^{\infty} {\rm d}z\,V_j^2\,(\,r\,)\,. \end{aligned} \tag{4.7}$$

The WKB phase thus contains the Glauber phase function $\delta_0^j(b)$ as its leading-order term in a derivative expansion in powers of the potential. Higher-order terms may be thought of as dynamical corrections to the Glauber straight-line trajectory inside the potential. Wallace further showed that the inclusion of higher-order corrections to the WKB approximation, as given by Rosen and Yennie [14], leads to an extra contribution which is linear in the potential and which cancels the unitarity corrections to the $S_F(b)$ operator.

5 The eikonal amplitude

Substituting the eikonal phases, defined for each of the potentials occurring in eq. (4.7)

$$\chi_{(c,l,s)}\,(b) = 2\delta_{(c,l,s)}^{\rm WKB}\,(b)\,, \tag{5.1}$$

into eq. (3.6) and retaining only the first-order term in eq. (4.3), we obtain

$$\begin{aligned} \bar{\chi}_0\,(b) &= \frac{1}{3}\,(\,3\chi_c\,(b) - 2\chi_s\,(b) + \chi_l\,(b))\,, \\ \Delta\chi_0\,(b) &= kb\,(\,2\chi_s\,(b) - \chi_l\,(b))\,, \\ D\chi_0\,(b) &= \chi_s\,(b) - \chi_l\,(b) + 2k^2b^2\chi_l\,(b)\,. \end{aligned} \tag{5.2}$$

These functions can now be inserted into the Fourier-Bessel representation of eq. (3.9) to arrive at Wallace's eikonalised scattering amplitudes. As expected, the resulting amplitudes reproduce the correct first Born approximation in the limiting case of a weak potential.

Noting that the operator in eq. (4.5) does not act on the combination kb, we define

$$\chi_{\gamma\beta}\,(b) = -\frac{\mu^2}{2k^3}\left\{1 + b\frac{\partial}{\partial b} - k\frac{\partial}{\partial k}\right\}\int_{-\infty}^{\infty} {\rm d}z\,V_\gamma\,(\,r\,)\,V_\beta\,(r)\,, \tag{5.3}$$

where $(\gamma, \beta) = (c, s, l)$. Using this together with eq. (4.7), we find the first quantum corrections:

$$\begin{aligned}
\bar{\chi}_1(b) &= \frac{2}{9}(kb)^4 \chi_{ll} + (kb)^2 \left(\frac{2}{3}\chi_{ss} - \frac{1}{18}\chi_{ll} - \frac{4}{9}\chi_{ls}\right) \\
&\quad + \chi_{cc} + \frac{1}{6}\chi_{ll} + \frac{1}{2}\chi_{ss} + \frac{2}{3}\chi_{cl} - \frac{4}{3}\chi_{cs} - \frac{5}{9}\chi_{ls}, \\
\Delta\chi_1(b) &= \frac{4}{3}(kb)^3 \left(\chi_{ls} - \frac{1}{2}\chi_{ll}\right) - (kb) \\
&\quad \times \left(2\chi_{ss} + \frac{1}{3}\chi_{ll} + 2\chi_{cl} - \frac{5}{3}\chi_{ls} - 4\chi_{cs}\right), \\
D\chi_1(b) &= -\frac{2}{3}(kb)^4 \chi_{ll} + (kb)^2 \left(2\chi_{ss} + \frac{5}{2}\chi_{ll} + 4\chi_{cl}\right. \\
&\quad \left. - \frac{16}{3}\chi_{ls}\right) - \frac{3}{2}\chi_{ss} - \frac{5}{6}\chi_{ll} - 2\chi_{cl} + \frac{7}{3}\chi_{ls} + 2\chi_{cs}.
\end{aligned} \tag{5.4}$$

Expanding the profile functions in eqs. (3.5), (3.7), and (3.8) in powers of the potential strength and retaining terms only up to second order, leads to

$$\begin{aligned}
S_F(b)\ \Gamma_A(b) &\sim \left\{\chi_c + \chi_{cc} + i\chi_c^2 + (kb)^2 \left(\frac{2}{3}\chi_{ss} + \frac{i}{3}\chi_s^2\right)\right. \\
&\quad \left. + \left(-5(kb)^2 + 4(kb)^4\right) \left[\frac{1}{18}\chi_{ll} + \frac{i}{36}\chi_l^2\right]\right\}, \\
S_F(b)\ \Gamma_B(b) &\sim \frac{1}{2\sin\theta} \left\{(kb)^2 \left(\frac{1}{2}\chi_s^2 - 2\chi_c\chi_s + 2i\chi_s\right.\right. \\
&\quad \left. - \frac{i}{2}\chi_{ss} + 2i\chi_{cs}\right) + \left(-\frac{1}{4} + 5(kb)^2 + 4(kb)^4\right) \\
&\quad \times \left.\left[\frac{i}{6}\chi_{ls} - \frac{i}{8}\chi_{ll} - \frac{1}{6}\chi_l\chi_s + \frac{1}{8}\chi_l^2\right]\right\}, \\
S_F(b)\ \Gamma_c^k(b) &\sim \frac{1}{\sin^2\theta} \left\{(kb)^2 \left(-\chi_l - \chi_{cl} + \frac{3}{2}\chi_{ls}\right.\right. \\
&\quad \left. - \frac{1}{2}\chi_{ss} - i\chi_c\chi_l + \frac{3i}{2}\chi_l\chi_s - \frac{i}{2}\chi_s^2\right) \\
&\quad \left. - \frac{1}{24}\left(\frac{1}{4} + 7(kb)^2 - 4(kb)^4\right) \left[\chi_{ll} + i\chi_l^2\right]\right\}.
\end{aligned} \tag{5.5}$$

The main difference between these spin-one expressions and the corresponding ones for the spin-zero and spin-half cases is that, due to the extra $b(kb)^2$ coefficient that appears in the spin-one profile functions, we here need to retain one term higher in the perturbative operator $S_F(b)$, namely

$$S_F(b) \sim \left[1 + \frac{1}{24k^2}\left(\frac{\partial}{\partial b}\right)^3 b\right]. \tag{5.6}$$

For example,

$$\Gamma_B(b)\,|_{\text{1st order}} \sim \frac{i}{\sin\theta}\left[(kb)^2 - \frac{1}{4}\right]\chi_s(b). \tag{5.7}$$

From the structure of the operator in eq. (3.4), it follows that the extra kb factor requires us to include the second term in the series for $S_F(b)$. To first order in the potential strength, the eikonal validity condition, $|k\chi(b)| \gg |\nabla\chi(b)|$, allows us to neglect all derivatives of the phase function arising from this operator; these are the unitarity corrections mentioned earlier. Hence we find

$$S_F(b)\ \Gamma_B(b)\,|_{\text{1st order}} \sim \frac{i(kb)^2}{\sin\theta}\chi_s(b). \tag{5.8}$$

Higher-order terms involve both kinematic (unitarity) and dynamical corrections, although the former are of the order $1/k$ smaller than the latter.

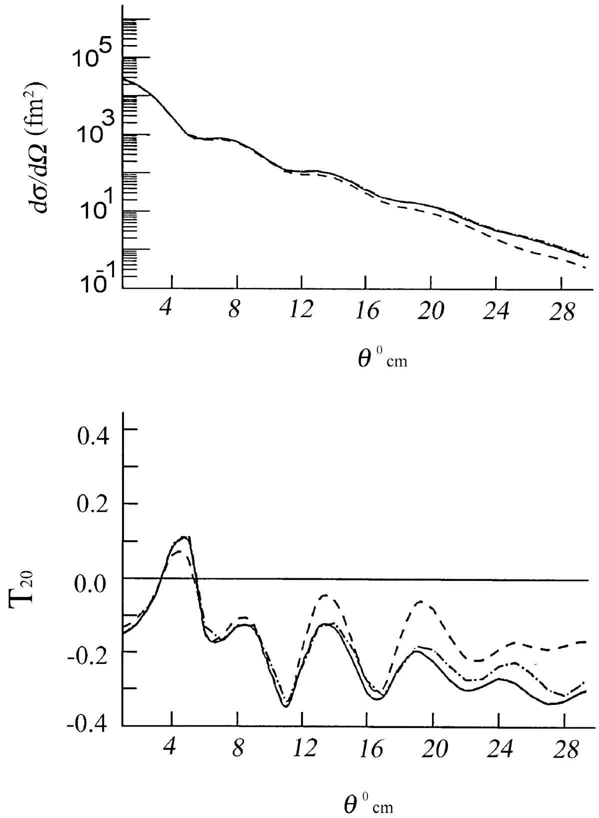

Fig. 1. Differential cross-section σ and tensor analysing power T_{20} for d-^{58}Ni elastic scattering at an incident deuteron energy of 400 MeV. The dot-dashed and the dashed curves represent the observables calculated using the eikonalised amplitudes with and without corrections, respectively. The solid curve represents a numerical resolution of the Schrödinger equation using the DDTP program [15].

6 Application to deuteron-nucleus elastic scattering

The deuteron is the simplest spin-one hadronic probe and innumerable experimental and theoretical studies of deuteron-nucleus elastic scattering have been undertaken. The deuteron internal structure reveals itself through the presence of a second-rank tensor contribution to the potential describing the d-A elastic amplitude. In this paper

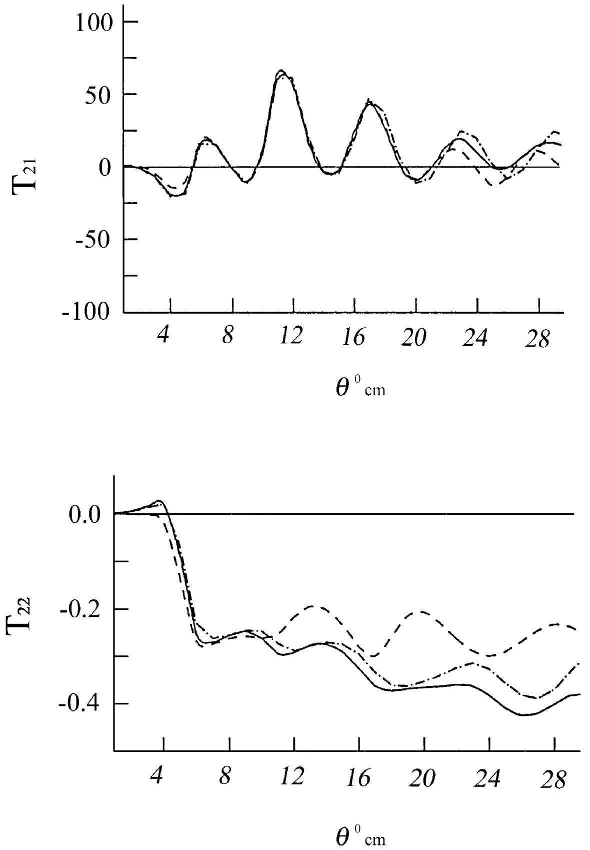

Fig. 2. Tensor analysing powers T_{21} ($\times 10^3$) and T_{22} for d-^{58}Ni elastic scattering at 400 MeV. The curves are as in fig. 1.

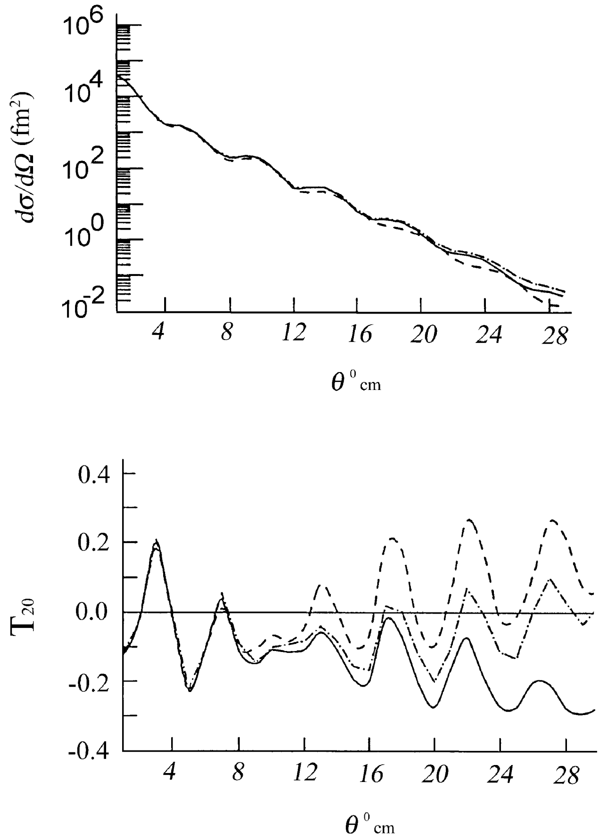

Fig. 3. Differential cross-section σ and tensor analysing power T_{20} for d-^{58}Ni elastic scattering at 700 MeV. The curves are as in fig. 1.

we have tested the formalism of the previous section in the case of d-^{58}Ni elastic scattering at incident deuteron energies of 400 and 700 MeV by comparing the results obtained with and without the eikonal corrections with those calculated numerically by solving the d-A two-body problem exactly via a partial-wave decomposition of the Schrödinger equation using the DDTP program [15]. In the Watanabe single-folding model [16], the d-A optical potential is obtained by folding the sum of V_p and V_n, evaluated at half the incident deuteron energy, over the deuteron ground-state wave function. At the incident deuteron energies of interest here (400 and 700 MeV), the nucleon-target interactions are therefore required to reproduce the appropriate nucleon-target scattering at 200 and 350 MeV, respectively. While nucleon data are not available for a ^{58}Ni target at these energies, phenomenological Dirac fits to nucleon elastic-scattering data give potential parameters that have little energy and target mass dependence and which can therefore be reliably interpolated to the target and energy required. For this reason V_p and V_n were taken as Schrödinger-equivalent reductions [17] of the global Dirac optical-potential fit of Hama *et al.* [18] to available nucleon scattering data (which included energies from 200 to 1000 MeV and targets of mass range 40–208). The folded deuteron potentials are evaluated in coordinate space using the techniques of Keaton and coworkers [19]. Early studies [20–22] of polarised deuteron scattering at these energies have shown that these folded potentials can, to a large extent, reproduce the elastic-scattering observables very well. For simplicity, we have assumed that the spin-orbit and the T_L tensor potentials have the same radial distribution.

The results for the differential cross-section and deuteron tensor and vector analysing powers are shown in figs. 1 to 5, where we compare the eikonal amplitudes (eq. (5.2), dashed curve) to the results including the first quantum corrections (eq. (5.4), dot-dashed curve) and the exact calculations (solid curves). The graphs clearly demonstrate that a significant improvement follows when the eikonal corrections are introduced, especially for the analyzing powers T_{20} and T_{22}. Nevertheless, there are still significant differences between the results of the corrected eikonal approach and those obtained from the DDTP program, more noticeably at larger angles and at higher energy, suggesting that higher-order quantum corrections are perhaps important here. The differences in the T_{21} and T_{11} analysing powers are smaller and those in the differential cross-section smaller still.

7 Discussion

Starting from the partial-wave representation, we have developed a generalised Fourier-Bessel expansion of the

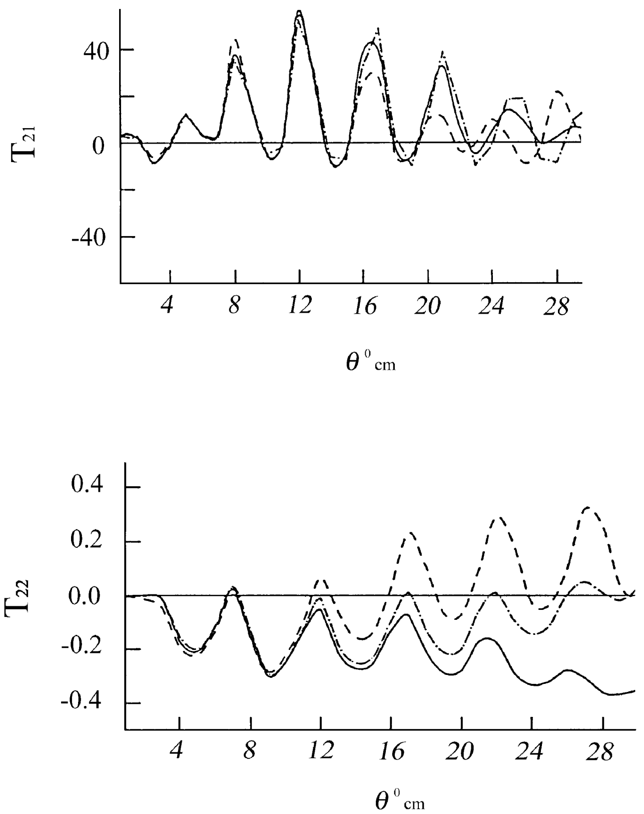

Fig. 4. Tensor analysing powers $T_{21}(\times 10^3)$ and T_{22} for d-^{58}Ni elastic scattering at 700 MeV. The curves are as in fig. 1.

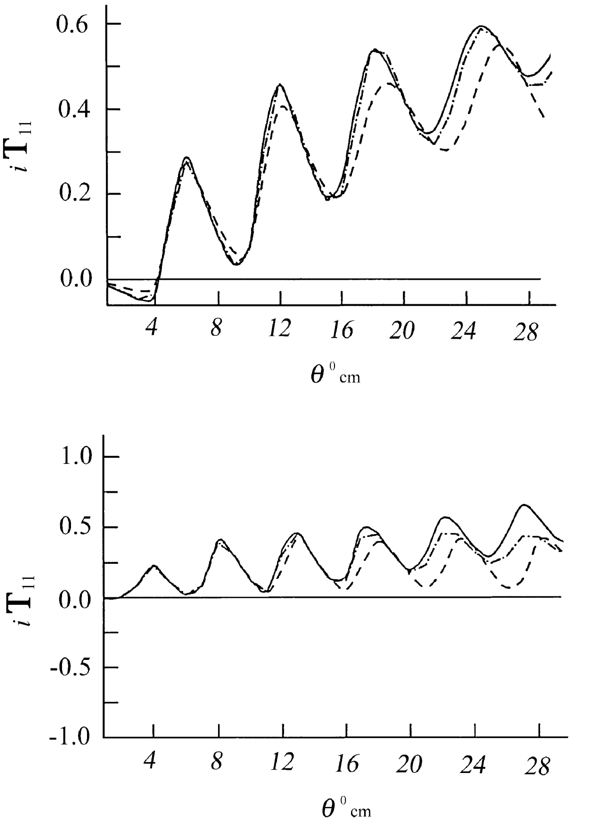

Fig. 5. The upper and lower graphs represent iT_{11} for d-^{58}Ni elastic scattering at 400 and 700 MeV, respectively. The curves are as in fig. 1.

amplitude for the scattering of a spin-one particle from a spinless target. In the spin-zero case, Wallace [5,7] has shown that the eikonal-phase function is the first term in an expansion of the WKB phase function in powers of the potential strength. The higher-order terms can be thought of as modifying the straight-line path of the particle in the potential and replacing it with the more realistic curved trajectory. These modifications recover the missing imaginary part of the second Born term and improve also the real part, leading to better agreement with the second Born term. Though we have studied the scattering of spin-one particles from a spinless target, the formalism could be modified to describe the central, spin-orbit and one of the spin-spin potentials for the scattering of two spin-half particles.

Byron *et al.* [23] have carried out a systematic study of the eikonal approximation, comparing the eikonal-Born series and the Wallace eikonal expansion. They report that, for a Yukawa potential in the weak-coupling limit, the eikonal approximation is consistently worse than the second Born approximation. Whereas, as the coupling is increased, the eikonal method becomes remarkably good at all angles. Their work also shows that the imaginary part of the Wallace amplitude improves on its eikonal counterpart and, in the large-angle limit, produces results that are more accurate than the imaginary part of the second Born amplitude.

Iseri *et al.* [24,25] have studied the spin-dependent interactions T_r and T_L for deuteron scattering and have demonstrated that a phenomenological T_L-type tensor interaction considerably improves the fit to the experimental data on the A_{yy} analysing power, while moderately improving A_y. Al Khalili *et al.* [26–28] have shown that the inclusion of coupling to the *np* singlet channel induces a T_L tensor interaction that has a considerable effect on the tensor analysing power A_{yy}.

Though there have been extensions of the Wallace approach for the scattering of spin-half particles [10,29], there seems to have been no previous attempt to investigate the higher-order terms of Wallace's expansion in the case of spin-one scattering, even for the limited choice of the tensor potential discussed here. We have compared our results with a numerical calculation employing plausible d-^{58}Ni potentials at 400 and at 700 MeV. The improvements from the corrections are very satisfactory and demonstrate clearly the need for the Wallace terms at intermediate energies. However, at large momentum transfers there are still discrepancies for T_{20} and T_{22}, suggesting that even higher-order corrections may be needed.

A more worrying problem is that there is no obvious way of extending our work to include the other two tensor potentials, which do not commute with the ones considered here; they mix states of different l. As a consequence, they do not exponentiate and it is hard to include them other than in an eikonal DWBA approach. Whether this is sufficient will, of course, depend upon the strength of such terms.

We wish to thank Professor R.C. Johnson for helpful comments.

Appendix A.

In this appendix we present, for completeness, observables for deuteron elastic scattering from a spin-zero nucleus in the Madison frame, which is defined such that the z-axis is along the direction of $\mathbf{k}_\mathrm{i} \times \mathbf{k}_\mathrm{f}$. In this frame the scattering matrix is of the form

$$M^m = \begin{pmatrix} A^m & B^m & C^m \\ D^m & E^m & -D^m \\ C^m & -B^m & A^m \end{pmatrix} . \qquad (A.1)$$

Only four of these amplitudes are independent, since they satisfy the relation

$$C^m = (A^m - E^m) - \sqrt{2}\,(B^m + D^m)\,\cot\theta . \qquad (A.2)$$

The scattering amplitudes are related to those defined by eq. (2.5) through

$$\begin{aligned}
A^m &= \frac{1}{3}\left[3A - C_n + C_k\left(3\cos^2\theta - 1\right)\right] ,\\
B^m &= -\frac{i}{\sqrt{2}}B + \frac{\sin\theta}{2\sqrt{2}}C_k ,\\
C^m &= -\frac{1}{2}\left[C_n - C_k\sin^2\theta\right] , \qquad (A.3)\\
D^m &= \frac{i}{\sqrt{2}}B + \frac{\sin\theta}{2\sqrt{2}}C_k ,\\
E^m &= A + \frac{1}{3}C_n + \frac{1}{3}C_k\left(3\cos^2\theta - 1\right) .
\end{aligned}$$

The differential cross-section and analysing powers in a spherical basis are then given by [30],

$$\begin{aligned}
3\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} &= 2|A_m|^2 + 2|B_m|^2 + 2|C_m|^2 + |D_m|^2 + |E_m|^2 ,\\
\sqrt{\frac{3}{2}}\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} iT_{11} &= \mathrm{Im}[B_m^*\,(A_m - C_m) + E_m^* D_m] ,\\
\frac{3}{\sqrt{2}}\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} T_{20} &= |A_m|^2 - 2|B_m|^2 + 2|C_m|^2 + |D_m|^2 - |E_m|^2 ,\\
\sqrt{\frac{3}{2}}\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} T_{21} &= -\mathrm{Re}[B_m^*\,(A_m - C_m) + E_m^* D_m] ,\\
\sqrt{3}\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} T_{22} &= 2\mathrm{Re}\,(A_m^* C_m) - |D_m|^2 .
\end{aligned} \qquad (A.4)$$

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